Assignment 11

1. Approximate a solution to the wave equation with four steps in time if the boundary conditions are $u_a(t) = \sin(t)$ and $u_b(t) = -\sin(t)$ and the initial states are $u_0(x) = \sin(\pi x)$ and $u_0^{(1)}(x) = 0$ if the interval in space is [0, 1] and $h = 0.2$. The wave speed is $c = 2$. You should use the Δt found in the course notes to ensure convergence.

2. What is c for electromagnetic waves? You may have to include an assumption.

3. Approximate a solution to Laplace's equation if we have a square region with sides $[0, 1] \times [0, 1]$ with $h = 0.25$ and two opposite walls are charged to 0 V and the other walls are at 5 V. What changes if the boundary point above the bottom right corner is changed to 100 V?

4. Approximate a solution to Laplace's equation if we have a square region with sides $[0, 1] \times [0, 1]$ with $h = 0.25$ and two opposite walls are charged to 0 V, one intermediate wall is at 5 V and the last wall is insulated.

5. How would you propose setting up the boundary conditions if you have a cross section of a coaxial cable that has a radius of 1.0 with an inner cable that has a radius of 0.4. The outside of the cable is kept at 0 V and the inner conducting cable is kept at 5 V. You should use an $h = 0.1$, but you don't have to explicitly set up the system of equations; instead, just indicate which points in the grid are associated with the 0 V boundary condition, which are associated with the 5 V boundary condition, and which are unknown and must be solved for.

6. Approximate a solution to Laplace's equation if we have a three-dimensional orthotope region with sides $[0, 1.25] \times [0, 1] \times [0, 1]$ with $h = 0.25$ and one square wall is at 0 V while the other five walls are insulated with the exception of a single point in the center of the opposite square wall that is kept at 5 V.

7. If you consider the solution to Question 6, you will realize that some points, by symmetry, must have the same value. Right now, we have a system of 36 equations in 36 unknowns. Use symmetry (identifying points that must have the same value) to reduce this to 12 equations in 12 unknowns.

8. Do the dimensions of the object change the result of Laplace's equation? For example, if the square region in Questions 3 and 4 were 1 cm or 1 km on each side, would this affect the approximations at the points, assuming h was also scaled appropriate?

9. Find an approximation for the minimum of the polynomial $x^4 + x^2 - 40x + 400$ using the step-by-step optimization routine described starting with $x_0 = 0$ and an initial $h = 1$, continuing until $h < 0.5$.

10. Find an approximation for the minimum of the polynomial $x^4 + x^2 - 40x + 400$ by applying two steps of Newton's method for finding extrema starting with $x_0 = 2$.

11. Find an approximation for the minimum of the function $\sin(x) - 2\sin(x) + \sin(2x)$ using the step-by-step optimization routine described starting with $x_0 = 0$ and an initial $h = 1$, continuing until $h < 0.5$.

12. Find an approximation for the minimum of the function $\sin(x) - 2\sin(x) + \sin(2x)$ by applying two steps of Newton's method for finding extrema starting with $x_0 = 2$.